7.2. Distinct, Real Characteristic Roots

Consider a constant coefficient, homogeneous, linear ODE of the form

$$a_2y'' + a_1y' + a_0y = 0. \qquad (*)$$

If the characteristic roots λ_1 and λ_2 are real and distinct, then the full solution of (*) consists of all functions of the form

$$y = c_1e^{\lambda_1 x} + c_2e^{\lambda_2 x},$$

where c_1 and c_2 are arbitrary constants. A fundamental set of solutions is given by

$$\{e^{\lambda_1 x}, e^{\lambda_2 x}\}.$$

Theorem 7.15 gives the nth order version of this.

Repeated Real Characteristic Roots

Let $\lambda \in \mathbb{R}$. A basis for the set of solutions to the ODE

$$y'' - 2\lambda y' + \lambda^2 y = 0$$

which can also be written as

$$(D - \lambda)^2(y) = 0$$

is given by the set

$$\mathcal{B} = \{e^{\lambda x}, xe^{\lambda x}\}.$$

Theorem 7.18 gives the nth order version of this.

Complex Characteristic Roots

Theorem 7.22: Suppose $at^2 + bt + c$ has non-real roots $\alpha \pm i\beta$. Then a basis of solutions to

$$ay'' + by' + cy = 0$$

is given by

$$\mathcal{B} = \{e^{\alpha x} \cos \beta x, e^{\alpha x} \sin \beta x\}.$$